

Course announcement

Interpretations of Gödel's theorems

(Vítězslav Švejdar, Fall 2021)

Gödel's paper [Göd31] is cited as the origin of Gödel's *first and second incompleteness theorems*. However, the second theorem is only implicit in that paper. Moreover, logic has evolved significantly since 1931. Nowadays, Gödel's theorems are formulated for extensions of one or another theory of natural numbers, without mentioning Principia Mathematica, and they are usually put into context with the notion of algorithm.

In the decade before Gödel published his paper, David Hilbert proposed a program known today as *Hilbert's program*. It emerged from several public talks given by Hilbert on various occasions, and its true meaning could probably still be the subject of considerations or research. One brief formulation could be “to put mathematics on secure ground by formalizing and investigating the notion of proof”, another is “to prove the consistency of arithmetic using finitary means”. There is a widely accepted view that Gödel's theorems refute Hilbert's program. However, some philosophers, a prominent example being Michael Detlefsen, think that it is not quite so, and they seem to have good reasons for that opinion.

The course will be attended by students of logic who already worked with Gödel's theorems, but other students interested in the history and philosophy of mathematics are also invited and very welcomed. If they show up, we can start with a brief introduction (one to three lessons) to logic, algorithmic aspects and the modern wording of Gödel's theorems. Besides this optional introductory part, the course will be based on students' presentations. Therefore, every participant is asked to contact me at vitezslav.svejdar@cuni.cz as soon as possible (already in September) and discuss her or his role in the whole business. Everybody should be ready to communicate in English. However, if all participants understand Czech, switching to Czech is possible. We want to read [Smo88] before everything else, and [Det79] and [Det90b] before other works by Detlefsen. The list of references below is not perfect. Suggesting some additions (ideally cited by Detlefsen) is welcomed. Nevertheless, Smoryński and Detlefsen should be of a supreme importance for us.

The course takes place Thursdays 14:10–15:40, starting from Oct 7th, room 137 in Celetná 20 (the room is located above the Blue Lecture Room and is accessed using the back staircase in Celetná 20). SIS ALGV19006, later also <http://www.cuni.cz/~svejdar/?s=igt>.

References

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- [Det07] M. Detlefsen. Formalism. In S. Shapiro, ed., *The Oxford Handbook of Philosophy of Mathematics and Logic*. Oxford University Press, 2007.
- [Det15] M. Detlefsen. Gödel's incompleteness theorems. In R. Audi, ed., *The Cambridge Dictionary of Philosophy*. Cambridge University Press, 2015. No pages etc., it is not clear how to cite this.
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